## CSCB58: Computer Organization



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The content of this lecture is adapted from the lectures of
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## CSCB58 Week 5

## We are here



## Circuits using flip-flops

- Now that we know about flip-flops and what they do, how do we use them in circuit design?

- What's the benefit in using flip-flops in a circuit at all?

We can design much cooler circuits using flip-flops.

## Example \#1: Registers

For storing values


## Shift registers

- A series of D flip-flops can store a multi-bit value (such as a 16bit integer, for example).

- Data can be shifted into this register one bit at a time, over 16 clock cycles.
- Known as a shift register.


## Load registers

- One can also load a register's values all at once, by feeding signals into each flip-flop:
- In this example: a 4-bit load register.


One clock pulse, 4 bits stored.

## Load registers

- To control when this register is allowed to load its values, we introduce the D flip-flop with enable:


When EN =1, D2 is whatever D is, load D
When $\mathrm{EN}=\mathrm{o}$, D 2 is whatever Q is, maintain Q

## Load registers



- Implementing the register with these special D flip-flops will now maintain values in the register until overwritten by setting EN high.

In computer architecture, registers are the CPU's most local storages (30+ of them on-chip), i.e., they are the lowest level of the memory hierarchy. They are the memory units that the CPU directly interact with.

Higher level of memory include cache, RAM, hard disc, etc.

Example \#2: Counters

## Counters

- Consider the T flip-flop:
- Output is inverted when input T is high.
- What happens when a series of T flip-flops are connected together in sequence?
- More interesting:
- Connect the output of one flip-flop to the clock input of the next!



## Counters

 not change upon the same clock signal.

- This is a 4-bit ripple counter, which is an example of an asynchronous circuit.
- Timing isn't quite synchronized with the rising clock pulse.
- Cheap to implement, but unreliable for timing.


## Demo: asynchronous human counter



1. Need 4 volunteers, O , $\mathrm{O}_{1}, \mathrm{O} 2, \mathrm{O} 3$
2. Qo toggles when receives clock signal (tap on shoulder)
3. O1 toggles when Qo goes from 1 to 0
4. Q2 toggles when Q1 goes from 1 to 0
5. O3 toggles when $\mathrm{O}_{2}$ goes from 1 to 0
6. Audience read the number $\mathrm{O}_{3} \mathrm{O}_{2} \mathrm{O}_{1} \mathrm{O} 0$

## Counters

- Timing diagram


- This is a synchronous counter, because all output Q's change upon the same clock edge.


## Counter with parallel load



- Counters are often implemented with a parallel load and clear inputs.
- Can set the counter to whatever value needed.

State Machines


## Designing with flip-flops

- Counters and registers are examples of how flip-flops can implement useful circuits that store values.

- How do you design these circuits?
- What would you design with these circuits?


## Designing with flip-flops

- Sequential circuits are the basis for memory, instruction processing, and any other operation that requires the circuit to remember past data values.
- These past data values are also called the states of the circuit.
- Need to describe the relation between the current state and the next state: use combinational circuits


## State example: Counters

- With counters, each state is the current number that is stored in the counter.


Each state does not need to
correspond to a binary number, they are just different states

- On each clock tick, the circuit transitions from one state to the next, based on the inputs.


## State Tables

- State tables help to illustrate how the states of the circuit change with various input values.
- Transitions are understood to take place on the clock ticks.

| State | Write | State |
| :---: | :---: | :---: |
| zero | 0 | zero |
| zero | 1 | one |
| one | 0 | one |
| one | 1 | two |
| two | 0 | two |
| two | 1 | three |
| three | 0 | three |
| three | 1 | four |
| four | 0 | four |
| four | 1 | five |
| five | 0 | five |
| five | 1 | six |
| six | 0 | six |
| six | 1 | seven |
| seven | 0 | seven |
| seven | 1 | zero |
|  |  |  |

## State Tables

- Same table as on the previous slide, but with the actual flip-flop values instead of state labels.
- Note: Flip-flop values are both inputs and outputs of the circuit here.

| $\mathbf{F}_{1}$ | $\mathbf{F}_{2}$ | $\mathbf{F}_{3}$ | Write | $\mathbf{F}_{1}$ | $\mathbf{F}_{2}$ | $\mathbf{F}_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |
|  |  |  |  |  |  |  |

and this brings us to...

## Finite State Machines

## Finite State Machines (FSMs)

- From theory courses...
- A Finite State Machine is an abstract model that captures the operation of a sequential circuit.
- A FSM is defined (in general) by:
- A finite set of states,
- A finite set of transitions between states, triggered by inputs to the state machine,
- Output values that are associated with each state or each transition (depending on the machine),
- Start and end states for the state machine.


## Design procedures comparison (roughly)

Combinational circuits

1. Desired behaviour
2. Truth table
3. Logic expression
4. Circuit

Sequential circuits

1. Desired behaviour (cooler behaviour)
2. Finite state machine
3. Circuit with flip-flops

## Example \#1: Tickle Me Elmo

- Remember how the Tickle Me Elmo works!



## Example \#1: Tickle Me Elmo

- Toy reacts differently each time it is squeezed:
- First squeeze $\rightarrow$ "Ha ha ha...that tickles."
- Second squeeze $\rightarrow$ "Ha ha ha...oh boy."
- Third squeeze $\rightarrow$ "HA HA HA HA...", go crazy
- Questions to ask:
- What are the inputs?
- What are the states of this machine?
- How do you change from one state to the next?


## Example \#1: Tickle Me Elmo



## Example \#2: Traffic Light



## FSM design

- Design steps for FSM:

1. Draw state diagram
2. Derive state table from state diagram
3. Assign flip-flop configuration to each state
4. Redraw state table with flip-flop values
5. Derive combinational circuit for output and for each flip-flop input.

## Example: Sequence Recognizer

- Recognize a sequence of input values, and raise a signal if that input has been seen.
- Example: Three high values in a row
- Detect that the input has been high for three rising clock edges.
- Assumes a single input X and a single output Z .


## What are the states?

## Step 1: State diagram

- In this case, the states are labeled with the most recent three input values.
- Transitions between states are indicated by the values on the transition arrows.



## Step 2: State table

- Make sure that the state table lists all the states in the state diagram, and all the possible inputs that can occur at that state.

| Previous <br> State | EN | Next <br> State |
| :---: | :---: | :---: |
| 000 | 0 | 000 |
| 000 | 1 | 001 |
| 001 | 0 | 010 |
| 001 | 1 | 011 |
| 010 | 0 | 100 |
| 010 | 1 | 101 |
| 011 | 0 | 110 |
| 011 | 1 | 111 |
| 100 | 0 | 000 |
| 100 | 1 | 001 |
| 101 | 0 | 010 |
| 101 | 1 | 011 |
| 110 | 0 | 100 |
| 110 | 1 | 101 |
| 111 | 0 | 110 |
| 111 | 1 | 111 |
|  |  |  |

## Step 3: Assign flip-flops

- Assign flip-flops for storing states.
- A single flip-flop can store two values (0 and 1), and thus two states.
- How many states can be stored with each additional flip-flop?
- One flip-flop $\rightarrow 2$ states
- Two flip-flops $\rightarrow 4$ states
- Three flip-flops $\rightarrow 8$ states
n states need: ceiling $\left(\log _{2} \mathrm{n}\right)$ flip-flops
- Eight flip-flops? $\rightarrow 2^{8}=256$ states


## How many flip-flops for this one?



3

## Step 3: Assign flip-flops

- In this case, we need to store 8 states.
- 8 states $=3$ flip-flops ( $3=\log _{2} 8$ )
- For now, assign a flip-flop to each digit of the state names in the FSM \& state table.



## Step 4: State table

- Mapping states to flip-flop values
- This is NOT the only way of mapping from state to flip flop values, in fact it is not even a good way, as we will see later.

| Prev. State | EN | Next State |
| :---: | :---: | :---: |
| 000 | 0 | 000 |
| 000 | 1 | 001 |
| 001 | 0 | 001 |
| 001 | 1 | 010 |
| 010 | 0 | 010 |
| 010 | 1 | 011 |
| 011 | 0 | 011 |
| 011 | 1 | 100 |
| 100 | 0 | 100 |
| 100 | 1 | 101 |
| 101 | 0 | 101 |
| 101 | 1 | 110 |
| 110 | 0 | 110 |
| 110 | 1 | 111 |
| 111 | 0 | 111 |
| 111 | 1 | 000 |

## Step 4: State table

- Mapping states to flip-flop values
- This is NOT the only way of mapping from state to flip flop values, in fact it is not even a good way, as we will see later.

| $\mathbf{F}_{\mathbf{2}}$ | $\mathbf{F}_{\mathbf{1}}$ | $\mathbf{F}_{0}$ | $\mathbf{E N}$ | $\mathbf{F}_{\mathbf{2}}$ | $\mathbf{F}_{\mathbf{1}}$ | $\mathbf{F}_{\mathbf{0}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

## Step 5: Circuit design

- Karnaugh map for $F_{2}$ :

|  | $\bar{F}_{0} \cdot \overline{\mathrm{EN}}$ | $\overline{\mathbf{F}}_{0} \cdot$ EN | $\mathbf{F}_{0} \cdot$ EN | $\mathbf{F}_{0} \cdot \overline{\mathrm{EN}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathbf{F}}_{2} \cdot \overline{\mathbf{F}}_{1}$ | 0 | 0 | 0 | 0 |
| $\bar{F}_{2} \cdot \mathbf{F}_{1}$ | 1 | 1 | 1 | 1 |
| $\mathrm{~F}_{2} \cdot \mathbf{F}_{1}$ | 1 | 1 | 1 | 1 |
| $\mathbf{F}_{2} \cdot \bar{F}_{1}$ | 0 | 0 | 0 | 0 |



## Step 5: Circuit design

- Karnaugh map for $\mathrm{F}_{1}$ :

|  | $\bar{F}_{0} \cdot \overline{\mathrm{EN}}$ | $\overline{\mathrm{F}}_{0} \cdot \mathrm{EN}$ | $\mathrm{F}_{0} \cdot$ EN | $\mathrm{F}_{0} \cdot \overline{\mathrm{EN}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathrm{F}}_{2} \cdot \overline{\mathrm{~F}}_{1}$ | 0 | 0 | 1 | 1 |
| $\bar{F}_{2} \cdot \mathrm{~F}_{1}$ | 0 | 0 | 1 | 1 |
| $\mathrm{~F}_{2} \cdot \mathbf{F}_{1}$ | 0 | 0 | 1 | 1 |
| $\mathrm{~F}_{2} \cdot \bar{F}_{1}$ | 0 | 0 | 1 | 1 |



## Step 5: Circuit design

- Karnaugh map for $F_{0}$ :

|  | $\bar{F}_{0} \cdot \overline{E N}$ | $\bar{F}_{0} \cdot$ EN | $F_{0} \cdot E N$ | $F_{0} \cdot \overline{E N}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\bar{F}_{2} \cdot \bar{F}_{1}$ | 0 | 1 | 1 | 0 |
| $\bar{F}_{2} \cdot F_{1}$ | 0 | 1 | 1 | 0 |
| $F_{2} \cdot F_{1}$ | 0 | 1 | 1 | 0 |
| $F_{2} \cdot \bar{F}_{1}$ | 0 | 1 | 1 | 0 |




## Step 5: Circuit design

- Resulting circuit looks like the diagram on the right.
- This will record the states and make the state transitions happen based on the input,
- What about the output value $Z$ which should go high when we have three highs in a row.



## Step 5: Circuit design

- Boolean equation for Z:

$$
Z=F_{0} \cdot F_{1} \cdot F_{2}
$$



## Moore machine vs Mealy machine

- Two ways to derive the circuitry needed for the output values of the state machine:
- Moore machine:
- The output for the FSM depends solely on the current state (based on entry actions).
- Mealy machine:
- The output for the FSM depends on the state and the input (based on input actions).
- Being in state X can result in different output, depending on the input that caused that state.


## An issue: timing and state assignments

- Example: if recognizer circuit is in state 011 and gets a o as an input, it moves to state 110.
- The first and last digits should change "at the same time", but they can't.
- If the first flip-flop changes first, the state will change to 111, and the output $Z$ would go high for an instant, which is unexpected behaviour.
- If the second flip-flop changes first, it's fine since the intermediate state 010 does NOT cause unexpected behaviour.


## An issue: timing and state assignments

- So how do you solve this?
- Two possible solutions:

1. Whenever possible, make flip-flop assignments such that neighbouring states differ by at most one flip-flop value.
" Intermediate states can be allowed if the output generated by those states is consistent with the output of the starting or destination states.
2. If the intermediate states are unused in the state diagram, you can set the output for these states to provide the output that you need.
" Might need to add more flip-flops to create these states.


State 000 does not have to have flip-flop values ooo, it can be anything you want to assign.

| Previous <br> State | EN | Next <br> State |
| :---: | :---: | :---: |
| 000 | 0 | 000 |
| 000 | 1 | 001 |
| 001 | 0 | 010 |
| 001 | 1 | 011 |
| 010 | 0 | 100 |
| 010 | 1 | 101 |
| 011 | 0 | 110 |
| 011 | 1 | 111 |
| 100 | 0 | 000 |
| 100 | 1 | 001 |
| 101 | 0 | 010 |
| 101 | 1 | 011 |
| 110 | 0 | 100 |
| 110 | 1 | 101 |
| 111 | 0 | 110 |
| 111 | 1 | 111 |

Home exercise: re-assign the states so the time issue doesn't exist

## After-class example: Mouse clicks

- Design a circuit that takes in two signals:
- A signal P, which is high if the user is pressing the mouse button,

- A signal $M$, which is high if the mouse is being moved.
- Based on the inputs, indicate whether the user is clicking, double-clicking, or dragging the mouse on the screen.

- Transitions indicate the values of P\&M.
- Outputs depend on the state (Moore machine)
- Home exercise: build the circuit for the mouse based on this FSM.

Next week
processor architecture

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